# GATE SCIENCE

# MATHEMATICS

## SOLVED SAMPLE PAPER

* DETAILED SOLUTIONS

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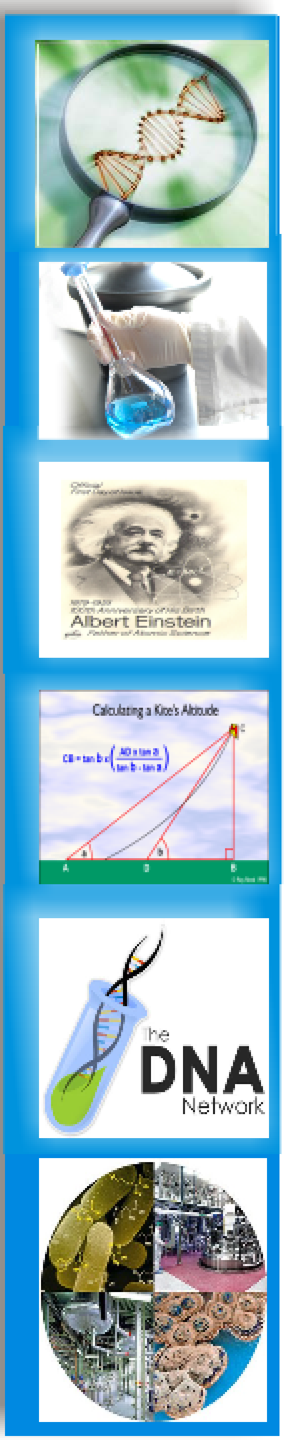


# GATE - MATHEMATICS

## MOCK TEST PAPER

- There are total of 65 questions in this paper which are of multiple choice type or numerical answer type.
- Questions Q.1 - Q.25 carry 1 mark each. Questions Q.26 - Q.55 carry 2 marks each. The 2 marks questions include two pairs of common data questions and two pairs of linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is not attempted, then the answer to the second question in the pair will not be evaluated.
- Questions Q. 56 - Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 - Q.60 carry 1 mark each, and questions Q. 61 - Q.65 carry 2 marks each.
- There will be negative marking of 1/3 marks for each wrong answer for 1 mark questions. For all 2 marks questions 2/3 marks will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question. There is no negative marking for questions of numerical answer type.

TIME : 3 HOURS

MAX. MARKS : 100

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**1.** Let f(x, y) = $\begin{cases} \frac{x^2y^2}{(x+y)^2}\{1+\log(1-(x+y))\}, & (x,y) \neq (0,0) \\ k, & (x,y) = (0,0) \end{cases}$

Then the value of k for which f(x, y) is continuous at (0, 0) is __________.

**2.** For what value of $\lambda$ the following matrix have nullity 1?

(A) $\lambda \neq 2$

(B) $\lambda \neq 3$

(C) $\lambda = 4$

(D) for all $\lambda$

**3.** If $3 \times 3$ is skew - Hermitian matrix and if have an eigen value –2i then one of the remaining Eigen values is _________.

**4.** Consider the l.p.p. Max z = $2x_1 - 4x_2$

s.t. $x_1 + 2x_2 \leq 3$

$3x_1 + 4x_2 \leq 5$

and $0 \leq x_1 \leq 5$

$0 \leq x_2 \leq 5$

the total no. of extreme points is ______.

**5.** Let $x^4$ be an optimal solution to the l.p.p.

minimize $c^Tx$

Subject to $Ax \geq b$

and $x \geq b$

which one of the following is true?

(A) The value of the objective function at a feasible solution to the dual L.P.P. is bounded above by $c^T x$.

(B) x is an extreme point of the feasible region

(C) The dual L.P.P. has an optimal solution with optimum value $c^T x$

(D) If a variable is zero in x then the corresponding constraint in the dual is satisfied as a strict inequality

**6.** The life time of two brands of bulbs X and Y are exponentially distributed with a mean life time of 100 hours. Bulb X is switched on 15 hours after bulb Y has been switched on. The probability that the bulb X fails before Y is

(A) $\frac{15}{100}$

(B) $\frac{1}{2}$

(C) $\frac{85}{100}$

(D) 0

**7.** Consider the algebraic extension $E = Q\left(\sqrt[3]{2}, \sqrt[4]{3}\right)$ of the field Q of rational numbers. Then [e : Q] the degree of E over Q is ________.

**8.** The general solution of the partial differential equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 12(x + y)$

(A) $(x + y)^2 + \phi(x + iy) + \phi_2(x - iy)$

(B) $(x + y)^3 + \phi_1(y + ix) + \phi_2(y - ix)$

(C) $(x + y)^3 + \phi_1(x + iy) + \phi_2(x - iy)$

(D) $(x + y)2 + \phi_1(y + ix) + \phi_2(y - ix)$

**9.** The numerical value obtained by applying the two point trapezoidal rule to the integral $\int_0^{\frac{\pi}{2}} e^{\sin x} dx$ is

(A) $\frac{1+e}{2}$

(B) $\frac{1-e}{2}$

(C) $\frac{1}{2}$

(D) $\frac{e-1}{2}$

**10.** Let f(x) be continuous whose values are known at –2, –1, 1, 2 if the Lagrange's interpolation formula $f(x) = L_1 f(-2) + L_2 f(-1) + L_3 f(1) + L_4 f(2)$ is used to approximate f(0) then $L_3$ is

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{4}{3}$

**11.** A steam boat is moving with velocity $v_1$ when steam is shut off. If the retardation at any subsequent time is equal to the magnitude of the velocity at that time, then the velocity fin time it after the steam in shut off is

(A) $v_1 e^t$

(B) $v_1 e^{-t}$

(C) $2v_1 e^t$

(D) $2v_1 e^{-t}$

**12.** Which one of the following statement is not correct?

(A) Every positive integer is either even or odd

(B) No integer is both even and odd

(C) For any integer, a, $a^2$ is even if and only if a is even

(D) No integer is both even and prime

**13.** Which one of the following is correct?

(A) Between any two rational numbers, there is an integer

(B) Between any two irrational numbers, there is a rational number

(C) Between any two irrational numbers, there is an integer

(D) Sum of two irrational numbers is always irrational

**14.** If f(x) and g(x) are differentiable function for $0 \le x \le 1$ such that f(1) - f(0) = k[g(1)-g(0)]. $k \ne 0$ then there exists c satisfying 0 < c < 1.

What is equal to $\frac{f'(c)}{g'(c)}$ ?

(A) 2k

(B) k

(C) –k

(D) $\frac{1}{k}$

**15.** Which one of the following functions is continuous at origin?

(A) $f(x) = \cos\left(\frac{1}{x}\right)$; when $x \ne 0$, f(0) = 0

(B) $f(x) = \sin x \sin\left(\frac{1}{x}\right)$; when $x \ne 0$, f(0) = 0

(C) $f(x) = x + \sin\left(\frac{1}{x}\right)$; when $x \neq 0$, $f(0) = 1$

(D) $f(x) = \sin x \sin\left(\frac{1}{x}\right)$, when $x \neq 0$, $f(0) = 1$

**16.** If $u = \frac{xy}{x+y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to:

(A) 1

(B) u

(C) – u

(D) 0

**17.** For the function $f(x,y) = x^5 F\left(\frac{y}{x}\right)$, the value of the differential $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to:

(A) $5x^5 F\left(\frac{y}{x}\right)$

(B) $5x^4 F\left(\frac{y}{x}\right)$

(C) $4x^5 F\left(\frac{y}{x}\right)$

(D) $4x^4 F\left(\frac{y}{x}\right)$

**18.** The set $S_1 = \left\{\alpha = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}, \beta = \begin{bmatrix} 2 & 4 & 8 \\ 6 & 0 & -2 \end{bmatrix}\right\}$ and $S_2 = \left(f = u^3 + 3u + 4, g = u^3 + 4u + 3\right)$ are :

(A) Both linearly dependent

(B) Both linearly independent

(C) $S_1$ is linearly dependent but $S_2$ is not

(D) $S_2$ is linearly dependent but $S_1$ is not

**19.** If the linear transformation T: $R^2 \to R^3$ is such that T (1, 0) = (2, 3, 1) and T(1, 1) = (3, 0, 2), then:

(A) T(x, y) = (x + y, 2x + y, 3x – 3y)

(B) T(x,y) = (2x + y, 3x – 3y, x + y)

(C) T(x, y) = (2x – y, 3x + 3y, x – y)

(D) T(x, y) = (x – y, 2x – y, 3x + 3y)

**20.** Multiplication of a complex number z by (1 + i) rotates the radius vector to z in the complex plane by an angle:

(A) 90° clockwise

(B) 45° clockwise

(C) 90° anticlockwise

(D) 45° anticlockwise

**21.** If $\left(\sqrt{3}+i\right)^{100} = 2^{99}(a+ib)$, then $a^2 + b^2$ is equal to _________.

**22.** In a city, three daily newspapers A, B, C are published, 42% of the people in that city read A, 51% read B and 68% read A and B; 28% read B and C; 36% read A and C; 8% do not read any of the three newspapers. The percentage of persons who read all the three papers is ___________.

**23.** If the set Z of integers is a group under the binary operation * defined as m * n = m + n + 1, n $\in$ Z, then the inverse of the element 5 is _________.

**24.** In an abelian group, the order of an element a is 4 and the order of an element b is 3, then $(ab)^{14}$ is equal to:

(A) $a^2b^{-1}$

(B) $(ab)^{-2}$

(C) $a^2$

(D) b

**25.** Which of the following rings are integral domains?

1. $Z_{60}$ 2. $Z_{71}$ 3. $Z_{82}$ 4. $Z_{97}$

Select the correct answer using the codes given below:

(A) 1 and 2

(B) 2 and 3

(C) 2 and 4

(D) 3 and 4

**26.** A ring (R, +, ·) whose all elements are idempotent is:

(A) Always abelian

(B) An integral domain

(C) A division ring

(D) A field

**27.** If $f(x)=\begin{cases} x^{\left(\frac{1}{k}\right)-1}\cos\left(\frac{1}{x}\right); & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ then under what condition is f9x) differentiable at x = 0?

(A) $k>\frac{1}{2}$

(B) $k\geq\frac{1}{2}$

(C) $k\leq\frac{1}{2}$

(D) $k<\frac{1}{2}$

**28.** Which one of the following differential equations is exact?

(A) $(3x^3 + 2y\sin 2x)\,d\,x + (2\sin 2x + 3y^3)dy = 0$

(B) $ye^{xy}dx + (e^{xy} + 2y)dy = 0$

(C) $(2\,xy\cos x^3 + 2xy + 1)\,dx + (\sin - x^2)\,dy = 0$

(D) $y\left(1+\frac{1}{x}\right)+\cos y+\left(x+\log x-x\sin y\right)\frac{dy}{dx}=0$

**29.** The general solution of the differential equation dy/dx = y/x + tany/x is:

(A) y = c x sin x

(B) y/x = sin x

(C) sin(y/x) = cx

(D) sin(y/x) = c sin x

where c is an arbitrary constant.

**30.** If integers a, b > 1; then the set of all integers of the form ma + nb (m, n integers) includes:

(A) Both their GCD and LCM

(B) Their GCD but not LCM

(C) Their LCM but not GCD

(D) Neither their LCM nor GCD

**31.** Which one of the following statements for sets A, B, C is correct?

(A) $A-(B\cup C)=(A-B)\cup(A-C)$

(B) $A\cup(B-C)=(A\cup B)-(A\cup C)$

(C) $A-(B\cap C)=(A-B)\cap(A-C)$

(D) $A-(B\cup C)=(A-B)\cap(A-C)$

**32.** If $u = x^2 + y^2 + z^2$, then

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$ is equal to:

(A) 4u

(B) $u^2$

(C) u

(D) 2u

**33.** If $T : R^3 \to R^3$, $T(x, y, z) = (x - y, y + 3z, x + 2y)$, then $T^{-1}$ is

(A) $\left(2x+z, -x+z, \frac{x}{3}+y-\frac{z}{3}\right)$

(B) $\frac{1}{3}\left(2x+y, -x+y, \frac{1}{3}x-\frac{1}{3}y+z\right)$

(C) $\frac{1}{3}\left(x+2y, x-y, -\frac{1}{3}x+\frac{1}{3}y-z\right)$

(D) $\frac{1}{3}\left(x-2y, x+y, \frac{x}{3}-\frac{y}{3}-z\right)$

**34.** Let V be a vector space of 2 × 2 matrices over R. Let T be the linear mapping $T : V \to V$, such that T(A) = AB – BA,

$B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, then the nullity of T is __________.

**35.** The value of $i^{1/3}$ are :

(A) $-i, \frac{i \pm \sqrt{3}}{2}$

(B) $i, \frac{i \pm \sqrt{3}}{2}$

(C) $-i, \frac{\sqrt{3}i \pm 1}{2}$

(D) $i, \frac{\sqrt{3}i \pm 1}{2}$

**36.** If $\alpha$ is a complex number such that $\alpha^2 + \alpha + 1 = 0$, then $\alpha^{31}$ is equal to __________.

**37.** If the matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} -i & 0 \\ 0 & 0 \end{pmatrix}$ from a group with respect to matrix multiplication, then which one of the following statements about the group, thus formed, is correct?

(A) The group has no element of order $4\alpha$

(B) The group has an element of order $3\alpha$

(C) The group is non-abelian

(D) $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$ is its own inverse

**38.** A random sample of size n is chosen from a population with probability density function f(x, $\theta$) = $\begin{cases} \frac{1}{2}e^{-(X-\theta)}, x \geq \theta \\ \frac{1}{2}e^{(x-\theta)}, x < \theta \end{cases}$ Then, the maximum likelihood estimator of $\theta$ is the

(A) Mean of the sample

(B) Standard deviation of the sample

(C) Median of the sample

(D) Maximum of the sample

**39.** The ring of integers (mod 6) is

(A) A finite integral domain

(B) An infinite integral domain

(C) A field

(D) Not an integral domain

**40.** If n denotes the number of elements in a field, then n must be:

(A) A prime

(B) A prime of the form 4k + 1

(C) A product of distinct primes

(D) A power of a prime

**41.** The singular solution of the differential equation

$(xp - y)^2 = p^2 - 1$, where $p = dy/dx$, is:

(A) $x^2 + y^2 = 1$

(B) $x^2 - y^2 = 1$

(C) $x^2 + 2y^2 = 1$

(D) $2x^2 + y^2 = 1$

**42.** The singularity of the equation

$= \frac{2}{3}x\frac{dy}{dx} - \frac{2}{3x}\left(\frac{dy}{dx}\right)^2, x > 0$ is:

(A) $y = x^3$

(B) $y = x$

(C) $y = x^3/6$

(D) $y = x^3/2$

**43.** The orthogonal trajectories of the system of parabolas $y^2 = 4a(x + a)$, a being the parameter, is given by the system of the curves:

(A) $y^2 = 4a(x + a)$

(B) $y^2 = 4a(x - a)$

(C) $y^2 = 4ax$

(D) $x^2 = 4ay$

**44.** Consider a mapping f from the set of natural numbers to the set of integers Z defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -n/2, & \text{when n is even} \end{cases}$$

Then $f: N \to Z$ is :

(A) one-one but not onto

(B) Onto but not one-one

(C) Both one-one and onto

(D) Neither one-one nor onto

**45.** If $\alpha$, $\beta$ are the roots of the equation $ax^2 + bx + c = 0$, then

$\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to:

(A) 0

(B) $\frac{a}{2}(\alpha - \beta)^2$

(C) $-\frac{a}{2}(\alpha - \beta)^2$

(D) $\frac{a^2}{2}(\alpha - \beta)^2$

**46.** If $f(x) = \begin{cases} x^\alpha \cos 1/x, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at x = 0, then:

(A) $\alpha < 0$

(B) $\alpha > 0$

(C) $\alpha = 0$

(D) $\alpha$ may b positive, negative or zero

**47.** The value of $\lim_{n \to \infty} \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$ is __________.

**Statement for COMMON DATA Questions 48 and 49**

Let a random variable X follow the exponential distribution with mean 2. Define Y = [X – 2| X > 2]

**48.** The value of $P(Y \geq t)$is

(A) $e^{-t/2}$

(B) $e^{-2t}$

(C) $\frac{1}{2}e^{-t/2}$

(D) $\frac{1}{2}e^{-t}$

**49.** The value of E(Y) is equal to ___________.

**Statement for COMMON DATA Questions 50 and 51**

Let the random variables X and Y be independent Poisson variates with parameters $\lambda_1$ and $\lambda_2$ respectively.

**50.** The conditional distribution of X given X+ Y is

(A) Poisson

(B) Hypergeometric

(C) Geometric

(D) Binomial

**51.** The regression equation of X on X + Y is given by

(A) $E(X \mid X+Y) = XY\frac{\lambda_1}{\lambda_1+\lambda_2}$

(B) $E(X \mid X+Y) = (X+Y)\frac{\lambda_2}{\lambda_1+\lambda_2}$

(C) $E(X \mid X+Y) = (X+Y)\frac{\lambda_1}{\lambda_1+\lambda_2}$

(D) $E(X \mid X+Y) = XY\frac{\lambda_2}{\lambda_1+\lambda_2}$

**Statement for Linked Answer Questions 52 and 53:**

Let f(z) = $\cos z - \frac{\sin z}{z}$ for non-zero $z \in C$ and f(0) = 0. Also, let g(z) = sinh z for $z \in C$

**52.** Then f(z) has a zero at z = 0 of order __________.

**53.** Then $\frac{g(z)}{zf(z)}$ has a pole at z = 0 of order __________.

**Statement for linked answer Questions 54 and 55**

Consider the boundary value problem

$u_{xx} = u_{yy} = 0,\ x \in (0, \pi),\ y \in (0, \pi),$

$u(x, 0) = u(x, \pi) = u(0, y) = 0.$

**54.** Any solution of this boundary value problem is of the form

(A) $\sum_{n=1}^{\infty} a_n$ sinh nx sin ny

(B) $\sum_{n=1}^{\infty} a_n$ cosh nx ny

(C) $\sum_{n=1}^{\infty} a_n$ sinh nx cos ny

(D) $\sum_{n=1}^{\infty} a_n$ cosh nx cos ny

**55.** If an additional boundary condition $u_x(\pi, y) = \sin y$ is satisfied, the $u(x, \pi/2)$ is equal to

(A) $\frac{\pi}{2}(e^{\pi} - e^{-\pi})(e^{\pi} - e^{-\pi})$

(B) $\frac{\pi(e^{x}+e^{-x})}{(e^{\pi}-e^{-\pi})}$

(C) $\frac{\pi(e^{x}+e^{-x})}{(e^{\pi}+e^{-\pi})}$

(D) $\frac{\pi}{2}(e^{\pi}+e^{-\pi})(e^{x}+e^{-x})$

**General Aptitude (GA) Questions (Q. 56-65)**

**56.** A butler stole wine from a butt of sherry, which contained 30% spirit and he replaced what he had stolen by wine containing 12% spirit. The butt was then of 18% strength. How much of the butt did he steal?

(A) 2/3

(B) 1/3

(C) 4/3

(D) 3/4

**57.** Find the average of all prime numbers between 30 and 50.

(A) 39.8

(B) 37.3

(C) 35

(D) 41

**58.** 64, 144, 256, 400...

(A) 529

(B) 484

(C) 676

(D) 576

**59.** What is the Antonym of Aggressive?

(A) Not getting justice

(B) Militant

(C) Retiring

(D) Noisy

**60.** What is the synonym of Voracious?

(A) Tenacious

(B) Truthful

(C) Spacious

(D) Ravenous

**61.** What is the synonym of Abortive

(A) Fruitful

(B) Familiar

(C) Unsuccessful

(D) Consuming

**62.** Fragile: Hardy

(A) Awkward: clumsy

(B) Orthodox: traditional

(C) Amateur: professional

(D) Cautious: flippant

**63.** Chapter: Book

(A) alcove: nook

(B) paragraph: sentence

(C) Page: rip

(D) room: house

**64.** What is the synonym of Bias

(A) Prejudice

(B) Tendency

(C) Resent

(D) Inclination

**65.** What is the Antonym of Coincidence?

(A) incidence

(B) Accident

(C) Chance

(D) Adaptation

# ANSWER KEY

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Answer. | 0 | B | 0 | 3 | C | B | 12 | B | D | B | B | D | B | B | B | B | A | C | B | D |
| Question | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| Answer. | 4 | 25 | -7 | A | C | C | D | D | C | B | D | D | A | 0 | A | 1 | D | D | D | A |
| Question | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| Answer. | B | C | A | A | D | B | 3 | A | 2 | D | C | 2 | 3 | A | C | A | A | D | C | D |
| Question | 61 | 62 | 63 | 64 | 65 | | | | | | | | | | | | | | | |
| Answer. | C | C | D | C | A | | | | | | | | | | | | | | | |

# HINTS AND SOLUTIONS

**1.** **0**

Given that f(x, y) is continuous at (0, 0) for that (x, y) $\to$ (0, 0) f(x, y) = f(0, 0)

$$\Rightarrow \quad \lim_{(x,y)\to(0,0)}\left[\frac{x^2y^2}{(x+y)^2}[1+\log(1-(x+y))\right] = k$$

$$\Rightarrow \quad \lim_{(x,y)\to(0,0)}\left[\frac{x^2y^2}{(x+y)^2}[1-(x+y)-\frac{(x+y)^2}{2!}.....\right] = k$$

$$\Rightarrow \quad 0 = k$$

**2. (B)** Since Let A = $\begin{bmatrix}1 & 1 & 1 & 6\\ 1 & 2 & 3 & 10\\ 1 & 2 & \lambda & \mu\end{bmatrix}$

by elementary matrix transformations changing A in echelon form.

$$A \sim \begin{bmatrix}1 & 1 & 1 & 6\\ 0 & 1 & 2 & 4\\ 0 & 1 & \lambda-1 & \mu-6\end{bmatrix} \sim \left[\begin{array}{ccc|c}1 & 1 & 1 & 6\\ 0 & 1 & 2 & 4\\ 0 & 0 & \lambda-3 & \mu-10\end{array}\right]$$

Rank (A) = 3 for $\lambda \neq 3$

$\Rightarrow$ nullity A = no. of columns – Rank A

= 4 – 3

= 1

**3.** **0**

A skew Hermitian matrix have all its eigenvalues either pure imaginary or zero and we know that imaginary values comes in pair so another eigenvalue of that matrix can be either 0 or 2i

**4.** **3**

Given l.p.p. Max $z = 2x_1 - 4x_2$

s.t. $x_1 + 2x_2 \leq 3$

$3x_1 + 4x_2 \leq 5$

$x_1 \leq 5$

$x_2 \leq 5$

here three extreme points but no. of constraint $\geq$ no. of variables so we get no. of basic feasible solution is more than no. of extreme points.

**5.(C)** If primal problem has an optimal solution then it's dual must have optimal solution and optimum values of primal and dual are same.

**6. (C)** Probability that the bulb X fails before Y = $\frac{100-15}{100} = \frac{85}{100}$

**7.** **12**

Clearly $[Q\left(\sqrt[3]{2},\sqrt[4]{3}\right) : Q] = Q[\left(\sqrt[3]{2},\sqrt[4]{3}\right) : Q\sqrt[3]{2}].[Q\left(\sqrt[3]{2}\right) : Q]$

By [K : F] = [K : E] [E : F]

and $[Q\left[\sqrt[3]{2},\sqrt[4]{3}\right] : Q] = [Q\left(\sqrt[3]{2},\sqrt[4]{3}\right) : Q\sqrt[4]{3}]\,[Q\left(\sqrt[4]{3}\right) : Q]$

Since $[Q\sqrt[3]{2} : Q] = 3$

and $[Q\sqrt[4]{3} : Q] = 4$

divide [Q$\left(\sqrt[3]{2},\sqrt[4]{3}\right)$ : Q] in both

Thus [Q $\sqrt[3]{2},\sqrt[4]{3}$ : Q] $\geq 0$

on the other hand [Q$\left(\sqrt[3]{2},\sqrt[4]{3}\right)$ : Q $\sqrt[3]{2}$ ] is at most 4 since $\sqrt[4]{3}$ is a zero for $x^4 - 3 \in Q\sqrt[3]{2}\,x$.

Therefore [Q$\left(\sqrt[3]{2},\sqrt[4]{3}\right)$: Q] = [Q $\sqrt[3]{2},\sqrt[4]{3}$ : Q $\sqrt[3]{2}$ ] [Q $\sqrt[3]{2}$ : Q]

$\leq 4.3$

$= 12$

**8. (B)** The given equation can be written as

$$(D^2 + D'^2) = 12(x + y) \qquad ...(1)$$

It's auxiliary equation is $m^2 + 1 = 0$

so that $m = \pm i$

C.F. = $\phi_1(y + ix) + \phi_2(y - ix)$

$\phi_1$ $\phi_2$ being arbitrary functions '

Now PI $= \dfrac{1}{D^2 + D'^2} 12(x + y)$

$= 12 \dfrac{1}{D^2 + D'^2} (x + y)$

$= \dfrac{12}{1^2 + 1^2} \int\int vdvdv$

$= 6 = v^3 = (x + y)^3$

Required GS $V = \phi_1(y + ix) + \phi_2(y - ix) + (x + y)^3$

**9.(D)** Two-point trapezoidal rule is

$$\int_a^b f(x)dx = \frac{f(b) - f(a)}{2}$$

Hence $\int_0^{\frac{\pi}{2}} e^{\sin x} dx = \dfrac{f\left(\frac{\pi}{2}\right) - f(0)}{2}$

$= \dfrac{e-1}{2}$

**10.(B)** $L_3$ is coefficient of term f(1) is given by

$$\frac{1}{(1+2)(1+1)(1-2)} = \frac{1}{3}$$

**11.(B)** A steam boat is moving with a velocity $v_1$ when steam is shut off. If the retardation at any subsequent time is equal to the magnitude of the velocity at the time, the velocity in time t after the steam is shut off will be equal to $v_1 e^{-t}$.

**12.(D)** No integer is both even and prime

Prime number-an integer $P \geq 2$ is said to be a prime number if its divisors are $\pm 1$ and $\pm P$.

$\therefore$ Integer 2 is prime and even both.

**13.(B)** Between any two irrational numbers, there is a rational number is a correct statement.

**14.(B)** By the Cauchy second mean value theorem we know that "if two function f, g defined on [a, b] are

1. continuous on [a, b]

2. derivable on (a, b) and

3. g'(x) $\neq$ 0 for any x $\in$ (a, b)

then there exists at let one real no. c between a and b such that

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

Now Here uses this theorem we get

$$\frac{f'(c)}{g'(c)} = k$$

**15.(B)** option-a f(x) = cos at $x \neq 0$ f(0) = 0

then $f(0+h) = \lim_{h\to 0} \text{coh}\,\frac{1}{h}$ = [Value b/w + to 1]

$f(0-h) = \lim_{h\to 0} \text{coh}\left(\frac{1}{-h}\right)$ = [Value b/w –1 to 1]

f(0) = 0

Since $(f0 + h) \neq f(0) \neq f(0 - h)$

Thus, f(x) is not continuous at x = 0

option-c $f(x) = x + \sin\frac{1}{x}$ at $x \neq 0$, f(0) = 1

then $f(0 + h) = \lim_{h\to 0} h + \sin\frac{1}{h}$

= 0 + [Value b/w – 1 to 1]

$f(0 - h) = \lim_{h\to 0} -h - \sin\left(\frac{1}{h}\right)$

= 0 – [Value b/w – 1 to 1]

Since $f(0th) \neq f(0) \neq f(0th)$

Thus f(x) is not continuous at x = 0

option -b

$f(x) = \sin x\frac{1}{x}$ sin at x = 0, f(0) = 0

then $f(0 + h) = \lim_{h\to 0} \sin h \sin\frac{1}{h}$

$= \sin o \sin\left(\frac{1}{0}\right)$

= 0

$f(0 - h) \quad = \lim_{h \to 0} \sin(-h) \sin\left(\frac{-1}{h}\right)$

$= \lim_{h \to 0} \sin h \frac{1}{h}$

$= 0$

Since f(0 + h) = f(0) = f(= – h)

Thus, the function is continuous at origin

**16.(B)** Here, it is given that

$$u = \frac{xy}{x + y}$$

$\because$ u is a homogeneous equation of degree one

$\therefore$ By Euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1 \cdot u = u$$

**17.(A)** Given function $f(x, y) = x^5 F\left(\frac{y}{x}\right)$

$\therefore$ We know Euler's theorem is

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y)$$

where $n \to$ degree of the function

$\therefore$ Using this theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 5x^5 F\left(\frac{y}{x}\right)$$

**18.(C)** Here, the sets

$$S_1 = \left\{\alpha = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}, \beta = \begin{bmatrix} 2 & 4 & 8 \\ 6 & 0 & -2 \end{bmatrix}\right\}$$

or $\quad S_1 = \left\{\alpha = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}, \beta = 2\begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}\right\}$

So, we can see that in set $S_1 \beta = 2\alpha$

$\therefore$ $S_1$ is linearly dependent.

and set

$$S_2 = \{f = u^3 + 3u + 4, g = u^3 + 4u + 3\}$$

then, $\quad f = u^3 + 3u + 4$

and $\quad g = u^3 + 4u + 3$

Now, $\quad af + bg = 0$

$\therefore \quad a(u^3 + 3u + 4) + b(u^3 + 4u + 3)$

or $\quad au^3 + 3au + 4a + bu^3 + 4bu + 3b = 0$

or $\quad (a + b)u^3 + (3a + 4b)u + (4a + 3b) = 0$

$\Rightarrow \quad a + b = 0$

$3a + 4b = 0$

and $\quad 4a + 3b = 0$

$\therefore$ From Eq. (i) $\quad a = -b$

Putting this value in Eq. (ii), we get

$3(-b) + 4b = 0$

or $\quad -3b + 4b = 0$

$\therefore \quad b = 0$

then $\quad a = 0$

and $\quad af + bg = 0 \Rightarrow a = 0$ and $b = 0$

Hence, f and g are linearly independent.

**19.(B)** First we all show that the set {(1, 0), (1, 1)} is a basis of $R^2$.

For linear independent of this set.

Let a(1, 0) + b(1, 1) = (0, 0) where $a, b \in R$

Then (a + b, b) = 0, 0

$\Rightarrow$ a + b = 0, b = 0

$\Rightarrow$ a = 0, b = 0

Hence, the set {(1, 0), (1, 1)} is linearly independent.

Now, we shall show that the set {(1, 0), (1, 1)} spans $R^2$. Let

$(x, y) \in R^2$ and Let(x, y) = a(1, 0) + b(1, 1)

(x, y) = (a + b, b)

$\Rightarrow$ a + b = x, b = y

$\Rightarrow$ a = x – b

a = x – b ($\because$ b = y)

or here $T(e_1) = (2, 3, 1)$

and $T(e_2) = (3, 0, 2)$

or set $(x, y) = ae_1 + be_2$

$T(x, y) = aT(e_1) + bT(e_2)$

= x – y(2, 3, 1) + y(3, 0, 2) = (2x – 2y + 3y), 3(x – y), (x – y + 2y)

T(x, y) = (2x + y, 3x – 3y, x + y)

**20.(D)** Arg z(1 + i) = Arg z + Arg (1+ i)

= Arg z + $\tan^{-1} 1$

= Arg z 45°

So, the multiplication of complex number z by (1 + i) rotates vector to z in the complex plane by an angle 45° anticlockwise.

**21.** **4**

Here, we will write

$$\left(\sqrt{3}+i\right)^{100} = 2^{100}\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{100}$$

When we change $\frac{\sqrt{3}}{2}+\frac{i}{2}$ into polar form, we get

Let $z = \frac{\sqrt{3}}{2}+\frac{i}{2} = r(\cos\theta + i\sin\theta)$

Then $r\cos\theta = \frac{\sqrt{3}}{2}$ and $r\ \sin\theta = \frac{1}{2}$

Now, squaring and adding these, we get

$$r^2\left(\cos^2\theta+\sin^2\theta\right) = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$r^2 = \frac{3}{4}+\frac{1}{4} = 1$$

or $r = 1$

$\therefore$ $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore$ The value of $\theta$, such that $-\pi < \theta < \pi$ and satisfying both the above equations is given by $\theta = \frac{\pi}{6}$.

$\therefore$ Required polar for $= \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$\therefore$ $\left(\sqrt{3}+i\right)^{100} = 2^{100}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{100}$

$$= 2^{100}\left(\cos\frac{100\pi}{6} + i\sin\frac{100\pi}{6}\right)$$

$$= 2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= 2^{100}\left\{\cos\left(8\times 2\pi + \frac{2\pi}{3}\right) + i\sin\left(8\times 2\pi + \frac{2\pi}{3}\right)\right\}$$

$$= 2^{100}\left\{\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right\}$$

$$\left(\sqrt{3} + I\right)^{100} = 2^{100}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2^{100}\left(-\frac{1+i\sqrt{3}}{2}\right)$$

$$= 2^{99}\left(-1 + i\sqrt{3}\right)$$

Comparing with $2^{99}(a+ib)$, we get

$$a = -1,\ b = \sqrt{3}$$

$$\therefore \quad a^2 + b^2 = (-1)^2 + \left(\sqrt{3}\right)^2$$

$$= 1 + 3 = 4$$

**Short method**

$$\frac{1}{2^{99}}\left(\sqrt{3} + i\right)^{100} = (a+ib)$$

$$\Rightarrow \quad 2\left[\frac{\sqrt{3}}{2} + \frac{i}{2}\right]^{100} = (a+ib)$$

$$\Rightarrow \quad 2\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right]^{100} = (a+ib)$$

$$\Rightarrow \quad 2\left[\cos\frac{100\pi}{6} + i\sin\left(\frac{100\pi}{6}\right)\right] = a+ib$$

$\Rightarrow \qquad a = 2\cos\frac{50\pi}{3}, b = 2\sin\left(\frac{50\pi}{3}\right)$

Now $a^2 + b^2 = 4$ **Ans.(D)**

**22.** **25**

Here, A, B, C are three published newspapers.

$\because$ Public read A newspaper n(A) = 42%

Public read B newspaper n(B) = 51%

Public read C newspaper n(C) = 68%

Public read newspaper A and B both $n(A \cap B)$ = 30%

Public read newspaper B and C both $n(B \cap C) = 28\%$

Public read newspaper A and C both $n(A \cap C) = 36\%$

Public who do not read any paper = 8%

Let the persons who read all the three papers = x

$$\therefore \qquad n(A \cup B \cup C) = n(A) + n(B) + n(A \cap B)$$

$$-n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

92 = 41 + 51 + 68 – 30 – 128 – 36 + x

or, 92 = 161 – 94 – 161 + 94

$\therefore$ x = 92 – 161 + 94

x = 25

**23.** **-7**

Here, It is given that the set Z of integers is group under the binary operation * defined as m * n = m + n + 1, m, n $\in$ Z.

$\therefore$ Identity element: let e be the identity element in Z for binary operation *, then

$m * e = m = e * m \ \forall m \in N$

$\Rightarrow \quad m * e = m$

$\Rightarrow \quad m + e + 1 = m$

$\Rightarrow \quad e = -1$

Inverse element: Let m be the inverse of the elements, then

$m * 5 = -1 = 5 * m$

$\Rightarrow \quad m + 5 + 1 = -1$

$\Rightarrow \quad m = -1 - 6 = -7$

**24.(A)** Here, it is given that in an abelian group, the order of an element a is 4 and the order of an element b is 3.

$\because$ Order of a is 4 $\therefore a^4 = e$

Where e is the identity of the group and order of b is 3

$\therefore \quad b^3 = e$

$\because$ Group is abelian, then

$(ab)^{14} = a^{14} \cdot b^{14}$

$= a^{12} \cdot a^2 \cdot b^{12} \cdot b^2$

$= (a^4) \cdot a^2 \cdot (b^3)^4 \cdot b^2$

$= e^3 \cdot a^2 \cdot e^4 \cdot b^2$

$= a^2b^2 \cdot bb^{-1}$

$= a^2b^3b^{-1}$

$= a^2eb^{-1}$

$= a^2b^{-1}$

**25.(C)** We know that the number of elements in a integral domain is either p or $p^n$.

$\therefore$ Only possible options are $Z_{71}$ and $Z_{97}$ containing prime number.

**26.(C)** A ring (R, +, ·) whose all elements are idempotent is a division ring.

**27.(D)** Given that the function f(9x) is

differentiable at x = 0 i.e.

$$f'(0) = \lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{(9h)^{\frac{1}{k}-1}\cos\left(\frac{1}{9h}\right)}{h}$$

$$= \lim_{h\to 0}\cos\left(\frac{1}{9h}\right)\lim_{h\to 0}9^{\left(\frac{1}{k}-1\right)}\,h^{\frac{1}{k}-2}$$

$$= [\text{value b/w} - 1 \text{ to } 1]\ 9^{\frac{1}{k}-1}\lim_{h\to 0}h^{\left(\frac{1}{k}-2\right)}$$

$$= 0 \text{ possible only when } \frac{1}{k}-2>0 \text{ i.e. } K<\frac{1}{2}$$

**28.(D)** We can check for exact differential equation

$$y\left(1+\frac{1}{x}\right)+\cos y+\left(x+\log x-x\sin y\right)\frac{dy}{dx}=0 \quad \text{... (i)}$$

It can be written as

$$\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+\{x+\log x-x\sin y\}dy=0$$

Here $M=y\left(1+\frac{1}{x}\right)+\cos y$

$\therefore$ $\frac{\partial M}{\partial y}=\left(1+\frac{1}{x}\right)-\sin y$

and $N = x + \log x - x\sin y$

$\therefore \quad \frac{\partial N}{\partial x} = \left(1 + \frac{1}{x}\right) - \sin y$

So, we can see that

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$$

$\therefore$ Differential equation is exact.

**29.(C)** Here, given differential equation is

$$\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x} \qquad \text{... (i)}$$

which is a homogeneous differential equation.

$\therefore$ Put $y = vx$

or, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{vx}{x} + \tan\frac{vx}{x}$

$\Rightarrow \quad v + x\frac{dv}{dx} = v + \tan v$

$\Rightarrow \quad x\frac{dv}{dx} = \tan v$

or, $\frac{dv}{\tan v} = \frac{dx}{x} \Rightarrow \cot v\, dv = \frac{dx}{x}$

Now, integrating it, we get

$\log \sin v = \log x + \log c$

$\Rightarrow \quad \sin v = x c$

$\Rightarrow \quad \sin\frac{y}{x} = c x$

**30.(B)** Here, it is given that integers a, b > 1

Let c be the GCD of a and b, then

$$c = (a, b)$$

If there exist integers m, n such that

$$c = ma + nb$$

then the set of all integers of the form ma + n (m, n integers) includes their GCD but not LCM.

**31.(D)** Let $x \in A - (B \cup C)$

$\Rightarrow$ $x \in A$ and $x \notin (x \cup C)$

$\Rightarrow$ $x \in A$ and ($x \in B$, $x \notin C$)

$\Rightarrow$ ($x \in A$ and $x \notin B$) and ($x \in A$ and $x \notin C$)

and $x \in (A - B)$ and $x \in (A - C)$

$\Rightarrow$ $x \in (A - B) \cap (A - C)$

$\therefore$ $A - (B \cup C) \subseteq A - (B \cup C)$

Similarly,

$$(A - B) \cap (A - C) \subseteq A - (B \cup C)$$

Hence, $A - (B \cup C) = (A - B) \cap (A - C)$

**32.(D)** Here $u = x^2 + y^2 + z^2$.

It is a homogeneous function of degree 2.

$\therefore$ By Euler's theorem $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu = 2u$

**33.(A)** Here, $T(x, y, z) = (x - y, y + 3z, x + 2y)$

Now, let $T(x, y, z) = (uvw)$

then $T^{-1}(uvw) = (x, y, z)$

So, $T(x, y, z) = (x - y, y + 3z, x + 2y) = (u, v, w)$

$\Rightarrow$ $x - y = u$ ....(i)

$y + 3z = v$ ....(ii)

$x + 2y = w$ ....(iii)

Taking equation (i) and (ii)

....(i)

....(ii)

By subtracting
$$\begin{array}{r} x - y = u \\ \underline{-x \pm 2y = -w} \\ 0 - 3y = u - w \end{array}$$

or $y = -\frac{u+w}{3}$

Putting this value in equation (i), we get

$$x - \left(\frac{-u+w}{3}\right) = u$$

or $x + \frac{u-w}{3} = u$

or $x = u - \frac{(u-w)}{3}$

$$x = \frac{3u-u+w}{3} = \frac{2u+w}{3}$$

and from equation (ii) $\left(\frac{-u+w}{3}\right) + 3z = v$

or $3z = v - \left(\frac{-u+w}{3}\right)$

$$3z = \frac{3v+u-w}{3}$$

$\therefore \quad z = \dfrac{3v+u-w}{3\times 3}$

$= \dfrac{3v}{9} + \dfrac{u}{9} - \dfrac{w}{9}$

$z = \dfrac{1}{3}\left(v+\dfrac{u}{3}-\dfrac{w}{3}\right)$

$\therefore \quad T^{-1}(u\ v\ w) = (x, y, z)$

or $\quad T^{-1}(u\ v\ w) = \dfrac{1}{3}(2u + w, -u + w, \dfrac{u}{3} + v - \dfrac{w}{3})$

or $\quad T^{-1}(x, y, z) = \dfrac{1}{3}(2x + z, -x + z, \dfrac{x}{3} + y - \dfrac{z}{3})$

**34.** **0**

Here linear mapping is $T : V \to V$, such that $T(A) = AB - BA$, where

$B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

Let $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then $\quad AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2a & a+3b \\ 2c & c+3d \end{bmatrix}$

and $\quad BA = \begin{bmatrix} 2a+c & 2b+d \\ 3c & 3d \end{bmatrix}$

$T(A) = AB - BA$

$= \begin{bmatrix} 2a & a+3b \\ 2c & c+3d \end{bmatrix} - \begin{bmatrix} 2a+c & 2b+d \\ 3c & 3d \end{bmatrix}$

$$T(A) = \begin{bmatrix} -c & a+b-d \\ -c & c \end{bmatrix} \sim \begin{bmatrix} -c & a+b-d \\ 0 & c-a-b+d \end{bmatrix}$$

i.e. The matrix of T has two non zero rows

Hence rank (T) = 2

$\Rightarrow$ nullity of T = 2 – 2 = 0

**35.(A)** Here, i1/3

Let $z = i/3$

$\Rightarrow$ $z^3 = (i^{1/3})^3 = i$

$\Rightarrow$ $z^3 - i = 0$

$\Rightarrow$ $z^3 + i \cdot i^2 = 0$

$\Rightarrow$ $z^3 + i^3 = 0$

$\Rightarrow$ $(z + i)(z^2 + i^2 - iz) = 0$ $[\because\ i^2 = -1]$

$\Rightarrow$ $z^2 + i^2 - iz = 0$

$\Rightarrow$ $z^2 - 1 - iz = 0$

$\Rightarrow$ $z^2 - iz - 1 = 0$

$\therefore$ $$z = -\frac{(-1) \pm \sqrt{(-i)^2 - 4 \cdot 1(-1)}}{2 \cdot 1}$$

$$= \frac{i \pm \sqrt{-1+4}}{2} = \frac{i \pm \sqrt{3}}{2}$$

$\therefore$ $z = -i, \dfrac{i \pm \sqrt{3}}{2}$

**36.** **1**

Here, given $\alpha$ is a complex number: $\alpha^2 + \alpha + 1 = 0$

$\therefore$ $\alpha^2 + \alpha + 1 = 0$, we will consider $\alpha = \omega$

$\therefore \quad \alpha^{31} = \omega^{31} = \omega^{31} \times 10 + 1 = \omega \qquad [\because \omega^3 = 1]$

**37.(D)** Identity element of the group is $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Now $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Hence, $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$ is its own inverse.

$\therefore$ This statement is correct. Ans. (D)

**38.(D)** Let $x_1, x_2, \ldots, x_n$ be a random sample of n observations from as population and probability density function is defined by

$$f(x, \theta) = \begin{cases} \frac{1}{2} e^{-(x-\theta)}, & x \ge \theta \\ \frac{1}{2} e^{(x-\theta)}, & x < \theta \end{cases}$$

it can be written as $f(x,\theta) = \frac{1}{2} e^{-|x-\theta|}$

The Likelihood function is defined by

$$L = \prod_{i=1}^{n} f(x_i, \theta) = \frac{1}{2^n} \exp\left[-\sum_{i=1}^{n} |x_i - \theta|\right]$$

$$L = \frac{1}{2^n} \exp\left[-n|\bar{x}_i - \theta|\right]$$

$$\log L = -2\log 2 - n\,|\bar{x}_i - \theta|$$

The Likelihood equations for estimating $\theta$. given

$$\frac{\partial}{\partial \theta} \log L = 0 = n$$

which is obviously inadmissible

Then we try to locate MLE' for $\theta$ by maximizing L directly

L is maximum ⇒ log L is maximum

Log L is maximum if ($\bar{x} - \theta$) is minimum which is so if $\theta$ maximum

if $x_{(1)}\ x_{(2)} \ldots\ x_{(n)}$ is ordered sample from

This population then

**39.(D)** Since, we know that the ring of integers modulo P is an integral domain if P is prime and if P is not prime. So, here ring of integers (mod 6) is not an integral domain.

**40.(A)** Given that n is the number of elements in a field, and we know that the ring of modulo ($Z_p$, $+_p$, $\bullet_p$) is a field if and only if p a prime (p = 2, 3, 5, 7, 9, ........)

i.e. order of the field is a prime no.

but, if n = 4k + 1 where $k \in N$ then

n = 5, 9, 13, ...... where 9 is not prime

i.e. n is not equal to a prime of the form 4k + 1 and also we know that no. of elements in a field is either prime or some power of prime.

**41.(B)** Here, given differential equation is

$$(xp - y)^2 = p^2 - 1$$

$$\Rightarrow \quad x^2p^2 + y^2 - 2xyp = p^2 - 1$$

$$\Rightarrow \quad (x^2 - 1)p^2 - 2xyp + y^2 + 1 = 0$$

which is quadratic in p.

∴ Singular solution will be

$$B^2 - 4A = 0$$

$$\Rightarrow \quad (-2xy)^2 - 4(x^2 - 1)(y^2 + 1) = 0$$

$$\Rightarrow \quad 4x^2y^2 - 4x^2y^2 + 4y^2 - 4x^2 + 4 = 0$$

$$\Rightarrow \quad 4y^2 - 4x^2 = -4$$

$\Rightarrow \quad 4x^2 - 4y^2 = 4s$

$\Rightarrow \quad x^2 - y^2 = 1$

**42.(C)** Here, given differentiation equation is

$$y = \frac{2}{3}x\frac{dy}{dx} - \frac{2}{3x}\left(\frac{dy}{dx}\right)^2, x > 0$$

or, $$y = \frac{2}{3}P - \frac{2}{3x}P^2 \qquad \left[\because P = \frac{dy}{dx}\right]$$

or, $$\frac{2}{3x}P^2 - \frac{2}{3}xP + y = 0$$

which is quadratic in P.

$\therefore$ Singular solution is given $B^2 - 4\,AC = 0$

$$\Rightarrow \left(\frac{2}{3}x\right)^2 - 4\left(\frac{2}{3x}\right)(y) = 0$$

$$\Rightarrow \frac{4}{9}x^2 - \frac{8y}{3x} = 0$$

$$\Rightarrow \frac{4}{9}x^2 = \frac{8y}{3x}$$

$$\Rightarrow \frac{x^2}{3} = \frac{2y}{x}$$

$$\Rightarrow x3 = 6y \text{ or, } y = \frac{x^3}{6}$$

**43.(A)** Here, given parabolas is

$$y^2 = 4a(x + a) \qquad \text{... (i)}$$

Now, differentiating it w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a$$

or, $y\frac{dy}{dx} = 2a$ ... (ii)

Now, eliminating a from Eqs. (i) and (ii), we get

$$y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} - y^2 = 0$$

or, $y^2p^2 + 2xP - y^2 = 0$

$\Rightarrow$ $yP^2 + 2xP - y = 0$

which is self orthogonal as by putting

$P = \frac{1}{P}$, we get the same equation

as $\Rightarrow$ $y\left(\frac{1}{P}\right)^2 + 2x\frac{1}{P} - y = 0$

$\Rightarrow$ $y \cdot \frac{1}{P^2} + \frac{2x}{P} - y = 0$

$\Rightarrow$ $y + 2xP - yP^2 = 0$

$\Rightarrow$ $yP^2 - 2xP - y = 0$

$\therefore$ The orthogonal trajectories of system of parabolas $y^2 = 4a(x + a)$ is itself.

i.e., $y^2 = 4a(x + a)$

**44.(A)** Here, it is given that the function $f : n \to Z$ defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -n/2, & \text{when n is even} \end{cases}$$

Let $n_1, n_2$ (odd) $\in$ N then $f(n_1) = f(n_2)$

$\Rightarrow$ $\frac{n_1 - 1}{2} = \frac{n_2 - 1}{2}$

$\Rightarrow$ $n_1 = n_2$

and $n_1, n_2$ (even) $\in$ N,

then $f(n_1) = f(n_2)$

$\Rightarrow$ $-\frac{n_1}{2} = -\frac{n_2}{2}$

$\Rightarrow$ $n_1 = n_2$

$\therefore$ f is one-one.

Again let t(odd) $\in$ Z, then

$$f(n) = t \Rightarrow \frac{n-1}{2} = t$$

$\Rightarrow$ n = 2t + 1

clearly n = 2t + 1 $\in$ N $\forall\, t \in Z$

and t(even) $\in$ Z, then

$$f(n) = t \Rightarrow \frac{-n}{2} = t$$

$\Rightarrow$ n = –2t

clearly $n = -2t \notin N \forall t \in Z.$

$\therefore$ f is not onto

$\therefore$ Hence, f is one-one but not onto.

**45.(D)** Here it is given that a, b are roots of the equation $ax^2 + bx + c = 0$

$\therefore$ $a + b - \frac{b}{a}$ = and $\alpha\beta = \frac{c}{a}$

$\therefore$ $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$

It is in $\frac{0}{0}$ form

So, by L'Hospital's rule

$$\lim_{x\to\alpha}\frac{(2ax+b)\sin\left(ax^2+bx+c\right)}{2(x-\alpha)}$$

$$\Rightarrow \quad \lim_{x\to\alpha}\frac{(2ax+b)^2\cos\left(ax^2+bx+c\right)+2a\sin\left(ax^2+bx+c\right)}{2}$$

$$\Rightarrow \quad \frac{(2a\alpha+b)^2}{2}$$

$$\Rightarrow \quad \frac{a^2}{2}\left(2\alpha+\frac{b}{a}\right)^2$$

$$\Rightarrow \quad \frac{a^2}{2}\left[2\alpha-(\alpha+\beta)\right]^2 \qquad \left[\because \alpha+\beta=\frac{-b}{a}\right]$$

$$\Rightarrow \quad \frac{a^2}{2}(\alpha-\beta)^2$$

**46.(B)** Here, it is given that

$$f(x)=\begin{cases} x^\alpha \cos 1/x, & x\neq 0 \\ 0, & x=0\end{cases}$$

is continuous at x = 0

$\therefore$ We have

$$R\lim_{\alpha\to 0} f(0+h)=\lim_{h\to 0} h^\alpha \cos\frac{1}{h}$$

$$=\lim_{h\to 0} h^{\alpha-1} h\cos\frac{1}{h}$$

= 0, If $\alpha-1\geq 0$ i.e., $\alpha\geq 1$ or $\alpha\geq 0$

Similarly,

$$R\lim_{h\to 9} f(0-h) = 0$$

and $f(0) = 0$

$\therefore$ f(x) is continuous at x = 0, if $\alpha > 0$

**47.** **3**

$$\lim_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+2^n} = \lim_{n\to\infty}\frac{\frac{3^{n+1}}{3^n}\left[(2/3)^{n+1}+1\right]}{\left[(2/3)^n+1\right]} = 3$$

**48.(A)** Probability distribution function of exponential distribution is given by

$$f(x, \theta) = \begin{cases}\theta e^{-\theta x} & x>0 \\ 0 & \text{otherwise}\end{cases}$$

and its mean = $\frac{1}{\theta}$

$\Rightarrow$ $\theta = \frac{1}{2}$

so pdf $f\left(x,\frac{1}{2}\right) = \begin{cases}\frac{1}{2}e^{-x/2} & x>0 \\ 0 & \text{otherwise}\end{cases}$

$P(Y \geq t) = \int_t^\infty f\left(x,\frac{1}{2}\right)dx = \int_t^\infty \frac{1}{2}e^{-x/2}dn = \frac{1}{2}\left[\frac{e^{-x/2}}{-Y_2}\right]_t^\infty = e^{-t/2}$ **ans.**

**49.** **2**

$$E(Y) = \frac{1}{2}\int_0^\infty xe^{-\frac{1}{2}x}dx$$

$$= \frac{1}{2}\left[\frac{xe^{-\frac{1}{2}x}}{-\frac{1}{2}} - 4e^{-x/2}\right]_0^\infty$$

$$= \frac{4}{2} = 2$$

**50.(D)** Let X and Y be independent poisson variates with parameters $\lambda_1$ and $\lambda_2$

Then X + Y is also poisson variate with parameter $\lambda_1 + \lambda_2$

$$p[X = r \mid (X + Y = n)] = \frac{p(X = r \cap X + Y = n)}{p(X + Y = n)}$$

$$= \frac{p(X = r \cap Y = n - r)}{p(X + Y = n)}$$

$$= \frac{p(X = r)p(X = n - r)}{p(X + Y = n)} \quad \text{[since X, Y independent ]}$$

$$= \frac{e^{-\lambda_1}\frac{\lambda_1^r}{r!}\cdot\frac{e^{-\lambda_2}(\lambda_2)^{n-r}}{(n-r)!}}{\frac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}{n!}}$$

$$= \frac{n!}{r!(n-r)!}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^r\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-r}$$

$$= {}^nc_r\, p^r q^{n-r}$$

where $p = \frac{\lambda_1}{\lambda_1+\lambda_2}$ $\qquad q = 1 - p$

Hence the conditional distribution of X given X + Y = n is a binomial distribution with parameters n and $p = \frac{\lambda_1}{\lambda_1+\lambda_2}$

**51.(C)** Regrssion equation of X on X + Y is given by

$$= \bar{X} + r\,\frac{\sigma_x}{\sigma_{x+y}}\left(x + y - \overline{x+y}\right)$$

Since X follows Poisson distribution with parameter $\lambda_1$ so mean $\bar{X} = \lambda_1$

SD = X = $\sqrt{\lambda_1}$

Also X + Y follows Poisson distribution with parameter $\lambda_1 + \lambda_2$

Then mean = $\lambda_1 + \lambda_2$

SD pf X + Y = $\sqrt{\lambda_1 + \lambda_2}$

and r is correlation coefficient between X and X + Y is given by

$$r = \frac{Cov(x, x+y)}{\sigma_x \sigma_{x+y}}$$

$$= \frac{Cov(x_1\ x) + cov(x, y)}{\sigma_x \sigma_{x+y}}$$

Since X, Y are independent

$\Rightarrow$ Cov(x, y) = 0

$$\Rightarrow \quad r = \frac{Cov(x, x)}{\sigma_x \sigma_{x+y}}$$

now $Cov(x_1\ x) = E(x^2) - (E(x))^2$

$= \lambda_1$

$$\text{so } r = \frac{\lambda_1}{\sqrt{\lambda_1}\sqrt{\lambda_1 + \lambda_2}}$$

$$\Rightarrow \quad \text{regression } = \lambda_1 + \frac{\lambda_1}{\sqrt{\lambda_1}\sqrt{\lambda_1 + \lambda_2}} \frac{\lambda_1}{\sqrt{\lambda_1 + \lambda_2}} (x + y(\lambda_1 + \lambda_2))$$

$$= \lambda_1 + \frac{\lambda_1}{\lambda_1 + \lambda_2} (x + y) - \frac{(\lambda_1 + \lambda_2)\lambda_1}{\lambda_1 + \lambda_2}$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2} (x + y)$$

Hence the answer.

**52.** **2**

Since $f(z) = \cos z - \frac{\sin z}{z}$

to find zeros of f(z) put f(z) = 0

$$\Rightarrow \qquad \cos z - \frac{\sin z}{z} = 0$$

$$\Rightarrow \qquad \left[1-\frac{z^2}{2!}+\frac{z^4}{4!}-.........\right]-\left[\frac{z-\frac{z^3}{3!}+\frac{z^5}{5!}-.....}{z}\right]=0$$

$$\Rightarrow \qquad \left(1-\frac{z^2}{2!}+\frac{z^4}{4!}........\right)-\left(1-\frac{z^2}{3!}+\frac{z^4}{5!}........\right)=0$$

$$\Rightarrow \qquad z^2\left[\frac{1}{3!}-\frac{1}{2!}\right]+........=0$$

so z = 0 is a zero of f(z) of order 2

**53.** **3**

$$\frac{g(z)}{zf(z)}=\frac{\sin hz}{z\cos z-\sin z}$$

To find poles z cos z – sin z = 0

$$\Rightarrow \qquad \left[z-\frac{z^3}{2!}+\frac{z^5}{4!}-.......\right] - \left[z-\frac{z^3}{3!}+.......\right]=0$$

$$\Rightarrow \qquad z^3\left[\frac{1}{3!}-\frac{1}{2!}\right]+z^5\left[\frac{1}{5!}+\frac{1}{4!}\right]=0$$

$\Rightarrow \qquad \frac{g(z)}{zf(z)}$ have pole at z = 0 of order 3

**54.(A)** Given $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ...(1)

it is a Laplace eq. of two dimensional let its's sol. is

$$U(x, y) = X(x)\,Y(y) \qquad ...(2)$$

then from (1) and (2)

$$\frac{1}{X_{(x)}}\frac{d^2X}{dx^2} + \frac{1}{Y(y)}\frac{d^2Y}{dy^2} = 0$$

$$\Rightarrow \qquad \frac{1}{X(x)}\frac{d^2X}{dx^2} = \frac{-1}{Y(y)}\frac{d^2Y}{dy^2}$$

Since L.H.S and R.H.S. both are independent to each other then each equal to same constant say $\lambda^2$ then

$$\frac{d^2X}{dx^2} = \lambda^2 X(x) \qquad ...(3)$$

and $$\frac{d^2Y}{dy^2} = -\lambda^2 Y(y) \qquad ...(4)$$

sol. of (3) $\quad X(x) = C_1\cosh\lambda x + C_2\sinh\lambda x \quad$ ...(5)

sol. of (4) $\quad Y(y) = d_1\cos\lambda y + d_2\sin\lambda y \quad$ ...(6)

Now using the boundary conditions.

$u(x, 0) = 0 \Rightarrow X(x)\,Y(0) = 0 \Rightarrow Y(0) = 0 \quad [X(x) \neq 0]$

$u(x, \pi) = 0 \Rightarrow X(x)\,Y(\pi) = 0 \Rightarrow Y(\pi) = 0 \quad [X(x) \neq 0]$

$u(0, Y) = 0 \Rightarrow X(0)\,Y(y) = 0 \Rightarrow X(0) = 0 \quad [Y(y) \neq 0]$

if $Y(0) = 0 \Rightarrow \boxed{d_1 = 0}$

if $Y(\pi) = 0 \Rightarrow d_2 \sin\lambda\pi = 0$

$\Rightarrow d_2 \neq 0 \quad \sin\lambda\pi = 0$

$$\lambda\pi = n\pi$$

$$\boxed{\lambda = n}$$

So $\boxed{Y_n(y) = d_n \sin ny}$ ...(7)

again $X(0) = 0 \Rightarrow$ $\boxed{C_1 = 0}$

So, $\boxed{X_n(x) = C_n \sinh nx}$ $[\lambda = n]$

Hence $U_n(x, y) = X_n(x) Y_n(y)$

$U_n(x, y) = C_n \sinh nx . d_n \sin ny$

So, $U_n(x, y) = \sum_{n=1}^{\infty} u_n(x, y)$

$= \sum_{n=1}^{\infty} a_n \sinh nx \sin ny$ $[a_n = c_n d_n]$

**55.(C)** Since $u_n(x, y) = \sum_n a_n \frac{e^{nx} - e^{-nx}}{2} \sin ny$

taking n = 1

$$a \cdot \left(\frac{e^x - e^{-x}}{2}\right) \sin y = u(xy)$$

$$u_x(x, y) = \frac{a}{2}(e^x + e^{-x}) \sin y$$

$$u_x(\pi, y) = \frac{a}{2}(e^\pi + e^{-\pi}) \sin y = \sin y$$

$$\Rightarrow \quad a_1 = \frac{2}{e^\pi + e^{-\pi}}$$

$$\text{so } u\left(x, \frac{\pi}{2}\right) = \frac{\cancel{2}}{e^\pi + e^{-\pi}} \frac{e^x - e^{-x}}{\cancel{2}} \sin\frac{\pi}{2} = \frac{e^x - e^{-x}}{e^\pi + e^{-\pi}}$$

Hence [C] is correct option.

**56.(A)** By the alligation rule, we find that wine containing 30% of spirit and wine containing 12% spirit should be mixed in the ration 1: 2 to produce a mixture containing 18% spirit.

This means that 1/3 of the butt of sherry was left; in other words, the butler drew out 2/3 of the butt.

Hence 2/3 of the butt was stolen.

**57.(A)** There are five prime number between 30 and 50.

They are 31, 37, 41, 43 and 47.

$\therefore$ Required average = $\frac{(31+37+41+43+47)}{5} = \frac{199}{5} = \mathbf{39.8}$

**58.(D)** The pattern is $8^2, 12^2, 16^2, 20^2,$

$\therefore$ Missing number = $24^2 = 576$.

**59.(C)** Pattern is .3, 3., 1.5. 1, 1.5. 15. 1.5, 2.25. 2, ....

$\therefore$ Missing number = 120.6 = 720.

**60.(D)** The meaning of Voracious is excessively greedy and grasping and the word ravenous provides the same meaning. So the synonym of Voracious is ravenous.

**61.(C)** The meaning of Abortive is Failing to accomplish an intended result and the word unsuccessful provides the same meaning. So the synonym of abortive is unsuccessful.

**62.(C)** Fragile is the opposite of hardy. Amateur is the opposite of professional.

**63.(D)** Chapters make up a book. Rooms make up a house.

**64.(A)** Bias means - A partiality that prevents objective consideration of an issue or situation.

and the word bias provides the same meaning. So the synonym of bias is prejudice.

**65.(C)** Antonym of Coincidence is incidence.